a)

$\langle \alpha' j' \| \vec{J} \| \alpha j \rangle$: $\langle \alpha' j' m' | T^k_q | \alpha j m \rangle = \langle jk, mq | jk; j'm' \rangle \dfrac{\langle \alpha' j' \| T^k \| \alpha j \rangle}{\sqrt{2j+1}}$

$$\Rightarrow J^{(1)}_0 = J_z \Rightarrow m\hbar\, \delta_{\alpha\alpha'} \delta_{jj'} \delta_{mm'} = \underbrace{\langle j1, m0 | j1, j'm' \rangle}_{\delta_{mm'} \langle j1, m0 | j1, j'm \rangle} \frac{\langle \alpha' j' \| J \| \alpha j \rangle}{\sqrt{2j+1}}$$

$$\Rightarrow m\hbar\, \delta_{\alpha\alpha'} \delta_{jj'} = \langle j1, m0 | j1, j'm \rangle \frac{\langle \alpha' j' \| \vec{J} \| \alpha j \rangle}{\sqrt{2j+1}}$$

$$\Rightarrow \langle \alpha' j' \| \vec{J} \| \alpha j \rangle = \delta_{\alpha\alpha'} \delta_{jj'} \langle \alpha j \| J \| \alpha j \rangle$$

$$\Rightarrow m\hbar = \langle j1, m0 | j1, jm \rangle \frac{\langle \alpha j \| J \| \alpha j \rangle}{\sqrt{2j+1}}$$

$m = j$ $\langle j1, m0 | j1, jj \rangle = \sqrt{\dfrac{j}{j+1}} \Rightarrow j\hbar = \sqrt{\dfrac{j}{j+1}} \dfrac{\langle \alpha j \| J \| \alpha j \rangle}{\sqrt{2j+1}}$

$$\boxed{\langle \alpha' j' \| \vec{J} \| \alpha j \rangle = \hbar \sqrt{j(j+1)(2j+1)}\, \delta_{\alpha\alpha'} \delta_{jj'}}$$

b)
$$\begin{cases} |jj\rangle |10\rangle = \alpha |jj\rangle + \beta |j+1, j\rangle \\ |j, j-1\rangle |11\rangle = \gamma |jj\rangle + \delta |j+1, j\rangle \end{cases}$$

J_+ : $\sqrt{2}\, |jj\rangle |11\rangle = \beta \sqrt{2(j+1)}\, |j+1, j+1\rangle \Rightarrow \beta = \dfrac{1}{\sqrt{j+1}}$

$\sqrt{2j}\, |jj\rangle |11\rangle = \delta \sqrt{2(j+1)}\, |j+1, j+1\rangle \Rightarrow \delta = \sqrt{\dfrac{j}{j+1}}$

$$\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \cos & \text{sen} \\ -\text{sen} & \cos \end{pmatrix} \Rightarrow \alpha = \delta \quad , \quad \boxed{\alpha = \sqrt{\frac{j}{j+1}}}$$

c) CCOC (S^2, S_z, S_{12}^2) $= (S_1 + S_2)^2 = S_1^2 + S_2^2 + 2 S_1 \cdot S_2 = \frac{3}{2}\hbar^2 + 2 S_1 \cdot S_2$

$$S^2 = (S_1 + S_2 + S_3)^2 = S_{12}^2 + S_3^2 + 2(S_1 + S_2) \cdot S_3 = S_{12}^2 + \frac{3}{4}\hbar^2 + 2(S_1 + S_2) \cdot S_3$$

$$H = \frac{A}{2\hbar^2}\left(S_{12}^2 - \frac{3}{2}\hbar^2 \right) + \frac{B}{2\hbar^2}\left(S^2 - S_{12}^2 - \frac{3}{4}\hbar^2 \right)$$

$S_{12} = 1, 0$

$S = \frac{3}{2}, \frac{1}{2}$

$1, \frac{3}{2}$: $\frac{A}{2}\left(2 - \frac{3}{2}\right) + \frac{B}{2}\left(\frac{15}{4} - 2 - \frac{3}{4}\right) = \frac{A}{4} + \frac{B}{2}$ deg 4

$1, \frac{1}{2}$: $\frac{A}{2}\left(2 - \frac{3}{2}\right) + \frac{B}{2}\left(\frac{3}{4} - 2 - \frac{3}{4}\right) = \frac{A}{4} - B$ 2

X $0, \frac{3}{2}$: $\frac{A}{2}\left(-\frac{3}{2}\right) + \frac{B}{2}\left(\frac{15}{4} - \frac{3}{4}\right) = -\frac{3A}{4} + \frac{3B}{2}$ X 4 X

$0, \frac{1}{2}$: $\frac{A}{2}\left(-\frac{3}{2}\right) + \frac{B}{2}\left(\frac{3}{4} - \frac{3}{4}\right) = -\frac{3A}{4}$ 2

① a) $\equiv$ simetría, ya que por rotaciones $\vec{r}_1^2, \vec{r}_2^2, \vec{r}_1 \cdot \vec{r}_2$ invariante $\Rightarrow$ no levanta degeneraciones sino que modifica todos los niveles por igual manteniéndose la deg en m

b) $\vec{R} = \frac{\vec{r}_1 + \vec{r}_2}{\sqrt{2}}$ $\quad \vec{r} = \frac{\vec{r}_1 - \vec{r}_2}{\sqrt{2}}$

$$H = -\frac{\hbar^2}{2m}\nabla_1^2 - \frac{\hbar^2}{2m}\nabla_2^2 + \frac{m\omega^2}{2}(r_1^2 + r_2^2) + \lambda(\vec{r}_1 - \vec{r}_2)^2$$

$$= -\frac{\hbar^2}{2m}(\nabla_R^2 + \nabla_r^2) + \frac{m\omega^2}{2}(R^2 + r^2) + 2\lambda r^2$$

$$= -\frac{\hbar^2}{2m}(\nabla_R^2 + \nabla_r^2) + \frac{m\omega^2}{2}R^2 + \left(\frac{m\omega^2}{2} + 2\lambda\right) r^2 \Rightarrow 2 \text{ osc. desacopl.}$$

$$E_\lambda = \hbar\omega\left(N_R + \tfrac{3}{2}\right) + \hbar\sqrt{\omega^2 + \frac{4\lambda}{m}}\left(N_r + \tfrac{3}{2}\right) \qquad \left(E_0 = 3(1 + \sqrt{1+2\lambda})\right)$$

$$E = \hbar\omega\left(N_1 + \tfrac{3}{2}\right) + \hbar\omega\left(N_2 + \tfrac{3}{2}\right)$$

c) HF, $\Psi = \varphi(r_1)\varphi(r_2)\frac{(\chi_1^+\chi_2^- - \chi_2^+\chi_1^-)}{\sqrt{2}}$; $\varphi_+ = \varphi\chi^+$, $\varphi_- = \varphi\chi^-$ orbitales $\Rightarrow h_1\varphi_1\chi^+ + \int dr_2 |\varphi(r_2)|^2 V(r_1 r_2)\{(1\,0)\binom{1}{0}\chi^+ + (0\,1)\binom{0}{1}\chi^+\}\varphi(r_1)\chi^+ - \int dr_2 |\varphi(r_2)|^2 V(r_1 r_2)\{(1\,0)\binom{1}{0}\varphi(r_1)\chi^+ + (0\,1)\binom{1}{0}\varphi(r_1)\chi^-\} = \epsilon\varphi(r_1)$

$$\Rightarrow \boxed{\left(-\frac{\hbar^2}{2\mu}\nabla_1^2 + \frac{m\omega^2 r_1^2}{2}\right)\varphi(r_1) + \int dr_2 |\varphi(r_2)|^2 V(r_1, r_2)\, \varphi(r_1) = \epsilon\varphi(r_1)}$$

idéntico para $\varphi(r_2)$

$\hbar = 2m = \omega/2 = 1$

$$\varphi(r) = \pi^{-3/4} e^{-r^2/2}$$

d) $\int d^3r' |\varphi(r')|^2 \lambda(r - r')^2 = 4\pi\lambda\int_0^\infty dr' r'^2 (r^2 + r'^2 - 2\underset{0}{\vec{r}\cdot\vec{r}'})\pi^{-3/2} e^{-r'^2} = \frac{4\pi\lambda}{\pi^{3/2}}\left(\frac{\sqrt{\pi}}{4}r^2 + 3\frac{\sqrt{\pi}}{8}\right)$

$$= \lambda\left(r^2 + \frac{3}{2}\right) \Rightarrow \left(-\frac{\hbar^2}{2m}\nabla_1^2 + \frac{m\omega^2}{2}r_1^2\right)\varphi(r_1) + \lambda\left(r^2 + \frac{3}{2}\right)\varphi(r_1) = \epsilon\varphi(r_1)$$

$$\Rightarrow \boxed{\left(-\frac{\hbar^2}{2m}\nabla_1^2 + \left(\frac{m\omega^2}{2} + \lambda\right) r_1^2\right)\varphi(r_1) = \left(\epsilon - \frac{3\lambda}{2}\right)\varphi(r_1)}$$

$\Rightarrow$ oscilador $\omega' = \sqrt{\omega^2 + \frac{2\lambda}{m}}$, $\epsilon' = \frac{3\hbar\omega'}{2} + \frac{3\lambda}{2} = \frac{3\hbar}{2}\sqrt{\omega^2 + \frac{2\lambda}{m}} + \frac{3\lambda}{2}$

ó $\epsilon' = 3\sqrt{1+\lambda} + \frac{3\lambda}{2}$, $\varphi'(r) = \pi^{-3/4}(1+\lambda)^{3/8} e^{-(1+\lambda)^{1/2} r^2/2}$

Iterando con φ' $\Rightarrow \varphi^2 = \varphi'$! y $\epsilon^2 = \frac{3}{2}\frac{2+3\lambda}{\sqrt{1+\lambda}}$ ~~...~~

$E_{HF} = 6\sqrt{1+\lambda}$ $\qquad$ ~~...~~

2-mcm-p2.jpg

(2) a) $u'' + \left(k^2 - \frac{l(l+1)}{r^2} - \frac{2mA}{\hbar^2 r^2}\right)u = 0 \qquad u(r) \sim j_\lambda(kr) \qquad \lambda(\lambda+1) = l(l+1) + \frac{2mA}{\hbar^2}$

$$(\lambda + 1/2)^2 = (l+1/2)^2 + \frac{2mA}{\hbar^2} \qquad \lambda + 1/2 = \sqrt{(l+1/2)^2 + \frac{2mA}{\hbar^2}} \quad , \ \lambda > 0 \ \forall l$$

$$-\frac{l\pi}{2} + \delta_l = -\frac{\lambda\pi}{2} \qquad \boxed{\delta_l = \frac{\pi}{2}(l-\lambda) = \frac{\pi}{2}\left(l + 1/2 - \sqrt{(l+1/2)^2 + \frac{2mA}{\hbar^2}}\right)} \quad \text{indep. } k!$$

b) $A > 0$ repulsivo $\delta_l < 0$

$A < 0$ atractivo $\delta_l > 0$

c) $f_k(\theta) = \frac{1}{k}\sum_l (2l+1)\, e^{i\delta_l} \operatorname{sen}\delta_l\, P_l(\cos\theta) \qquad \sigma(\theta) = |f_k(\theta)|^2$

$$|\delta_l| \ll 1 \qquad f_k(\theta) \simeq \frac{1}{k}\sum_l (2l+1)\,\delta_l\, P_l(\cos\theta) \ , \ \delta_l \simeq \frac{\pi}{2}\left[\cancel{(l+1/2)} - \cancel{(l+1/2)}\left(\cancel{1} + \frac{2mA}{2\hbar^2(l+1/2)^2}\right)\right]$$

$$\boxed{\delta_l = -\frac{\pi m A}{2\hbar^2 (l+1/2)}} \qquad f_k(\theta) \simeq \frac{1}{k}\sum_l \left(-\frac{\pi m A}{\cancel{2}\hbar^2}\right)\frac{1}{\cancel{l+1/2}}\,\cancel{2}\cancel{(l+1/2)}\,P_l(\cos\theta)$$

$$f_k(\theta) \simeq -\frac{\pi m A}{\hbar^2 k}\sum_l P_l(\cos\theta) = -\frac{\pi m A}{2\hbar^2 k}\,\frac{1}{\operatorname{sen}\theta/2}$$

$$\boxed{\sigma(\theta) = \frac{\pi^2 m^2 A^2}{4\hbar^4 k^2}\,\frac{1}{\operatorname{sen}^2\theta/2}}$$

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