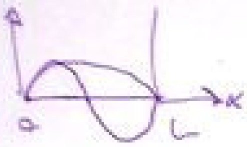
2)

a) $$\bar{D}^2 = \langle (x_1-x_2)^2 \rangle = \begin{cases} \int_0^L dx_1 \int_0^L dx_2\, (x_1-x_2)^2 \left(\frac{\psi_n(x_1)\psi_m(x_2) \pm \psi_n(x_2)\psi_m(x_1)}{\sqrt{2}} \right)^2 & n \neq m \\ \int_0^L dx_1 \int_0^L dx_2\, (x_1-x_2)^2\, \psi_n^2(x_1)\, \psi_n^2(x_2) & n = m \end{cases}$$



(spin f.o. antisim/sim según sean fermiones o bosones)

$$\psi_n(x) = \sqrt{\frac{2}{L}} \operatorname{sen} n\pi x/L \; ; \quad \int_0^L \psi_n^2(x)\,dx = \frac{2}{L}\int_0^L \operatorname{sen}^2 n\pi x/L \, dx = 1$$

$\underline{n=m}$ Tanto para B como F, f.o. espacial simétrica

$$\bar{D}^2 = \int_0^L dx_1 \int_0^L dx_2 \left(x_1^2 + x_2^2 - 2x_1x_2\right) \psi_n^2(x_1)\, \psi_n^2(x_2) = 2\langle x^2\rangle_n - 2\left(\langle x\rangle_n\right)^2$$

$$\langle x\rangle_n = \frac{L}{2} = \int_0^L dx\; x\, \frac{2}{L} \operatorname{sen}^2 n\pi x/L = \int_0^1 \frac{2}{L} L^2\, du\; \underbrace{u \operatorname{sen}^2 n\pi u}_{} = \frac{L}{2}$$

$$\langle x^2\rangle_n = \frac{2}{L} L^3 \int_0^1 dx\; x^2 \operatorname{sen}^2 n\pi x = 2L^2 \frac{1}{2}\left(\frac{1}{3} - \frac{1}{2n^2\pi^2}\right) = L^2\left(\frac{1}{3} - \frac{1}{2n^2\pi^2}\right)$$

$$\Rightarrow \bar{D}^2 = 2\left[L^2\right]\left[\frac{1}{3} - \frac{1}{2n^2\pi^2} - \frac{1}{4}\right] = L^2\left(\frac{1}{6} - \frac{1}{n^2\pi^2}\right) \Rightarrow \boxed{\bar{D} = L\sqrt{1/6 - 1/n^2\pi^2}} \; F, B$$

$\underline{n \neq m}$

$\Rightarrow$ F.o. espacial Sim o Asim según sean B o F, y según la f.o. espín.

$$\bar{D}^2 = \frac{1}{2}\int_0^L dx_1 \int_0^L dx_2 \left(x_1^2 + x_2^2 - 2x_1x_2\right)\left(\psi_n^2(x_1)\,\psi_m^2(x_2) + \psi_n^2(x_2)\,\psi_m^2(x_1) \pm 2\psi_n(x_1)\psi_m(x_1)\psi_n(x_2)\psi_m(x_2)\right)$$

$$= \frac{1}{2}\left[\left(\langle x^2\rangle_n + \langle x^2\rangle_m - 2\langle x\rangle_n \langle x\rangle_m\right) + (\text{idem}) \pm 2(-2)\left(\langle x\rangle_{nm}\right)^2\right] \; ; \text{ obs: } n, m \text{ ortogonales si } n \neq m.$$

$$\langle x\rangle_{nm} = \frac{2}{L}\int_0^L dx\; x \operatorname{sen} n\pi x/L \operatorname{sen} m\pi x/L = 2L\left(-\cancel{2}nm\right)\frac{1}{\pi^2(m^2-n^2)^2}\frac{\left(1+(-)^{n+m}\right)}{\cancel{2}}$$

$$\bar{D}^2 = \frac{L^2}{2}\left[\frac{2}{3} - \frac{2}{2n^2\pi^2} + \frac{2}{3} - \frac{2}{2m^2\pi^2} - 2\left(\frac{1}{2}\right)^2 2 \mp 4(-1)^2 \frac{4n^2m^2}{\pi^4(m^2-n^2)^4}\left(1+(-1)^{n+m}\right)^2\right]$$

$$= L^2\left(\frac{1}{6} - \frac{1}{2\pi^2}\left(\frac{1}{n^2} + \frac{1}{m^2}\right) \mp \frac{8n^2m^2}{\pi^4(m^2-n^2)^4}\left(1+(-)^{n+m}\right)^2\right)$$

$\Rightarrow n, m =$ paridad

$$\boxed{\bar{D} = L\left(\frac{1}{6} - \frac{1}{2\pi^2}\left(\frac{1}{n^2} + \frac{1}{m^2}\right) \mp \frac{32\, n^2m^2}{\pi^4(n^2-m^2)^4}\right)^{1/2}}$$

S, A

b) $s = 1/2$ F singulete $\Rightarrow$ Asim $\Rightarrow$ F.O. esp Sim $\Rightarrow$ signo $-$; $\Rightarrow$ mayor distancia en tripl. que en singulete $(n \neq m)$.

triplete $\Rightarrow$ Sim $\Rightarrow$ Asim $\Rightarrow$ $+$

Si $n = m$ $\Rightarrow$ singulete únicamente

$\underline{n=1\;\; m=2}$ $\bar{D} = L\left(1/6 - \frac{1}{2\pi^2}(1 + 1/4)\right)^{1/2} = L\left(1/6 - 5/8\pi^2\right)^{1/2}$

$$\left(\quad \right) \mp \frac{32 \cdot 4}{\quad} \Big)^{1/2} = L\left[\left(1/6 - \frac{5}{8\pi^2}\right) \mp \frac{128}{81\pi^4}\right]^{1/2} = L\left(0.103 \mp 0.016\right)^{1/2}$$

a) $|jm\rangle \xrightarrow{R} \left(\sum_{m'} |jm'\rangle\langle jm'|\right) \mathcal{D}(R)|jm\rangle = \sum_{m'} \langle jm'|\mathcal{D}(R)|jm\rangle \, |jm'\rangle$

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$$= \sum_{m'} \mathcal{D}^{(j)}_{m'm}(R) \, |jm'\rangle$$

b) $T^k_q|0\rangle \longrightarrow \mathcal{D}(R)\, T^k_q |0\rangle = \underbrace{\mathcal{D}(R)\, T^k_q\, \mathcal{D}^\dagger(R)}_{\sum_{q'} \mathcal{D}^{(k)}_{q'q}(R)\, T^k_{q'}} \underbrace{\mathcal{D}(R)|0\rangle}_{\text{invariante},\ |0\rangle}$

$$= \sum_{q'} \mathcal{D}^{(k)}_{q'q}(R)\, T^k_{q'}|0\rangle \Rightarrow \text{transforma como } |kq\rangle$$

c) $\langle \alpha l_1 || Y^l || \alpha l_2\rangle$:

WE : $\underbrace{\langle \alpha' l_1 0 | Y_{l0} | \alpha l_2 0\rangle}_{\text{CG series}^*} = \langle l_2 l; 0 0 | l_2 l; l_1 0\rangle \dfrac{\langle \alpha' l_1 || Y_l || \alpha l_2\rangle}{\sqrt{2l_2+1}}$

$$= \sqrt{\frac{(2l+1)(2l_2+1)}{4\pi(2l_1+1)}}\, \langle l_2 l; 00 | l_1 0\rangle \langle l_2 l; 00 | l_2 l; l_1 0\rangle$$

$$\Rightarrow \boxed{\langle \alpha' l_1 || Y_l || \alpha l_2\rangle = (2l_2+1)\sqrt{\frac{2l+1}{4\pi(2l_1+1)}}\, \langle l_2 l; 00 | l_2 l; l_1 0\rangle}$$

* $\langle \alpha' l_1 0 | Y_{l0} | \alpha l_2 0\rangle = \int d\Omega\, Y^*_{l_1 0}\, Y_{l0}\, Y_{l_2 0} \Rightarrow$ CG series.

p1-par1.jpg

1

a) Es op. vectorial : conmuta con $[\vec{S}, \vec{L}] = 0 \Rightarrow [S_i, J_j] = [S_i, S_j] = i\epsilon_{ijk} S_k$

b) $\vec{\sigma}\cdot\vec{\sigma}$ rango cero y prop. a $I_{2\times 2}$

rango 1 : $\sigma^{(1)}_\alpha$: $\sigma^{(1)}_0 = \sigma^3$, $\sigma^{(1)}_1$ $\sigma^{(1)}_{-1}$ los cof. corresp.

c) $\langle J_+ \| \vec{\sigma} \| J_+ \rangle$: $J_+ = l + \frac{1}{2}$

$$\frac{\langle J_+ \| \sigma^{(1)} \| J_+ \rangle}{\sqrt{2J_+ + 1}} \cdot \langle J_+ 1 ; m 0 | J_+ 1 ; J_+ m' \rangle = \langle J_+ m | \sigma^{(1)}_0 | J_+ m \rangle$$

$$\Rightarrow \frac{\langle J_+ \| \sigma \| J_+ \rangle}{\sqrt{2J_+ + 1}} \cdot \underbrace{\langle J_+ 1 ; J_+ 0 | J_+ 1 ; J_+ J_+ \rangle}_{= \left(\frac{J_+}{J_+ + 1}\right)^{1/2}} = \langle J_+ J_+ | \sigma^{(1)}_0 | J_+ J_+ \rangle$$

$$|J_+ J_+\rangle = |l + \tfrac{1}{2}, l + \tfrac{1}{2}\rangle = |l l\rangle |\tfrac{1}{2} \tfrac{1}{2}\rangle$$

$$\Rightarrow \sigma^{(1)}_0 |J_+ J_+\rangle = |J_+ J_+\rangle \Rightarrow \langle J_+ J_+ | \sigma^{(1)}_0 | J_+ J_+ \rangle = 1$$

$$\Rightarrow \langle + \| \sigma \| + \rangle = \sqrt{\frac{(2J_+ + 1)(J_+ + 1)}{J_+}} = \sqrt{\frac{(2l+2)(l + \frac{3}{2})}{l + \frac{1}{2}}} = \boxed{\sqrt{\frac{(2l+2)(2l+3)}{2l+1}} = \langle + \| \sigma \| + \rangle}$$

$\langle J_- \| \sigma \| J_- \rangle$:
$$\langle J_- J_- | \sigma^{(1)}_0 | J_- J_- \rangle = \underbrace{\langle J_- 1 ; J_- 0 | J_- 1 ; J_- J_- \rangle}_{= \left(\frac{J_-}{J_- + 1}\right)^{1/2}} \frac{\langle J_- \| \sigma \| J_- \rangle}{\sqrt{2J_- + 1}}$$

$$|J_- J_-\rangle = A |l, l\rangle |\tfrac{1}{2} -\tfrac{1}{2}\rangle + B |l, l-1\rangle |\tfrac{1}{2} \tfrac{1}{2}\rangle$$

de clase/Sakurai
$$A = -\sqrt{\frac{l + (l - \frac{1}{2}) + \frac{1}{2}}{2l+1}} = -\sqrt{\frac{2l}{2l+1}}$$
$$B = \sqrt{\frac{l - (l - \frac{1}{2}) + \frac{1}{2}}{2l+1}} = \sqrt{\frac{1}{2l+1}}$$

$$\sigma^{(1)}_0 |J_- J_-\rangle = \sigma^3 |J_- J_-\rangle = -A |l l\rangle |\tfrac{1}{2} -\tfrac{1}{2}\rangle + B |l, l-1\rangle |\tfrac{1}{2} \tfrac{1}{2}\rangle$$

$$\Rightarrow \langle J_- J_- | \sigma^{(1)}_0 | J_- J_- \rangle = B^2 - A^2 = \frac{1}{2l+1} - \frac{2l}{2l+1} = \frac{1-2l}{2l+1} = -\frac{J_-}{J_- + 1}$$

$$\Rightarrow \langle J_- \| \sigma \| J_- \rangle = -\frac{J_-}{J_- + 1} \sqrt{\frac{J_- + 1}{J_-}} \sqrt{2J_- + 1} = -\sqrt{\frac{J_- (2J_- + 1)}{J_- + 1}} = -\sqrt{\frac{(l - \frac{1}{2})(2(l - \frac{1}{2}) + 1)}{l - \frac{1}{2} + 1}}$$

$$\boxed{\langle - \| \sigma \| - \rangle = -\sqrt{\frac{2l(2l-1)}{2l+1}}}$$